Mathematics 122 **Fall 2002**

Exam 4

November 19, 2002

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Name

Technology used:____ Directions:

Be sure to include in-line citations, including page numbers if appropriate, every time you use a text or notes or technology. Include a careful sketch of any graph obtained by technology in solving a problem. **Only write on one side of each page.**

The Problems

1. (5, 15 points) Do **both** of the following.

   (a) Write the following improper integral as the sum of integrals having exactly one 'impropriety'. **Do not solve.**

   $$\int_{-\infty}^{\infty} \frac{e^x}{x\,(x-1)}\,dx$$

   (b) Evaluate the following integral

   $$\int \frac{9x+21}{(x-1)(x^2+4)}\,dx$$

2. (20 points) Use the $\varepsilon - N$ definition of limit to prove

   $$\lim_{x\to\infty} \frac{2x+1}{x+1} = 2$$

3. (10, 10 points) Compute any **two** (2) of the following limits. Include work to justify your answers.

   (a)

   $$\lim_{x\ \to\infty} x^2 \sin(\frac{1}{x})$$

   (b)

   $$\text{show } \lim_{n\ \to\infty} c^{\frac{1}{n}} = 1, \text{ where } c \text{ is a positive constant}$$

   (c)

   $$\lim_{x\to\infty} \frac{\sqrt{x^2+1}}{\sqrt{4x^2-3x}}$$

4. (10, 10 points) For any **two** (2) of the following improper integrals and infinite series, determine if they are convergent or divergent? If convergent, find their limit.

   (a)

   $$\int_{-\infty}^{0} \frac{1}{x^2+1}\,dx$$

(b)

$$\sum_{k=2}^{\infty}(k+1)^{\underline{-3}}$$

(c)

$$\sum_{k=2}^{\infty}\left(\frac{5}{11}\right)^{k+3}$$

5. Do **one** (1) of the following.

   (a) (15, 5 points) The function $G(m)=\frac{1}{2}\sum_{k=0}^{\infty}k^{\underline{m-1}}2^{-k}$ is a sequential function analogous to the Gamma function of the last quiz. The domain of this function is $m=1,2,3,\cdots$.

   i. Use Discrete Integration by Parts to show that

   $$G(m+1)=m\,G(m)$$

   ii. Given the fact that $G(1)=1$, in a few sentences, explain why $G(m+1)=m!$

   (b) (10, 10 points) Evaluate **both** of the following.

   i.

   $$\sum_{k=2}^{\infty}\left[\frac{1}{k}-\frac{1}{k+3}\right]$$

   ii.

   $$\int\frac{1}{x(x+1)(x-2)}\,dx$$

Useful Information about Sequences

$D_k\left[k^{\underline{n}}\right]=nk^{\underline{n-1}}$		$\frac{d}{dx}\left[x^n\right]=nx^{n-1}$
$D_k\left[k^{\underline{-n}}\right]=-n(k+1)^{\underline{-n-1}}$		$\frac{d}{dx}\left[x^{-n}\right]=-nx^{-n-1}$
$D_k\left[c^k\right]=(c-1)\,c^k$		$\frac{d}{dx}\left[c^x\right]=\ln(c)\,c^x$
$D_k\left[A(k)\right]=a(k)\rightarrow\sum a(k)=A(k)+C$		$\frac{d}{dx}\left[F(x)\right]=f(x)\rightarrow\int f(x)\,dx=F(x)+C$
$\sum k^{\underline{n}}=\frac{1}{n+1}k^{\underline{n+1}}+C$		$\int x^n dx=\frac{1}{n+1}x^{n+1}+C$
$\sum k^{\underline{-n}}=\frac{1}{-n+1}(k-1)^{\underline{-n+1}}+C$, if $n\neq 1$		$\int x^{-n}dx=\frac{1}{-n+1}x^{-n+1}+C$, if $n\neq 1$
$\sum\frac{1}{k^{\underline{1}}}=?$		$\int\frac{1}{x}dx=\ln\lvert x\rvert+C$
$\sum c^k=\frac{1}{c-1}c^k+Q,\ \ c\neq 1$		$\int c^x dx=\frac{1}{\ln(c)}c^x+Q,\ \ c\neq 1$
$\sum 1^k=k+C$		$\int 1dx=x+C$
$\sum_{k=0}^{n}a(k)=A(k)\big\rvert_0^{n+1}=A(n+1)-A(0)$		$\int_a^b f(x)\,dx=F(x)\Big\rvert_a^b=F(b)-F(a)$
$\sum_{k=0}^{n}U_k\,v_k=U_kV_k\big\rvert_0^{n+1}-\sum_{k=0}^{n}V_{k+1}u_k$		$\int_a^b u\,dv=uv\Big\rvert_a^b-\int_a^b v\,du$
$\sum_{k=r}^{\infty}a(k)=\lim_{n\rightarrow\infty}\sum_{k=r}^{n}a(k)$		$\int_a^{\infty}f(x)\,dx=\lim_{b\rightarrow\infty}\int_a^b f(x)\,dx$
$0\leq a(k)\leq b(k)$ and $\sum_{k=r}^{\infty}b(k)$ conv. $\implies\sum_{k=r}^{\infty}a(k)$ conv.		$0\leq f(x)\leq g(x)$ and $\int_a^{\infty}g(x)\,dx$ conv. $\implies\int_a^{\infty}f(x)\,dx$ conv.
$0\leq a(k)\leq b(k)$ and $\sum_{k=r}^{\infty}a(k)$ div. $\implies\sum_{k=r}^{\infty}b(k)$ div.		$0\leq f(x)\leq g(x)$ and $\int_a^{\infty}f(x)\,dx$ div. $\implies\int_a^{\infty}g(x)\,dx$ div.